Technical Comments

Comment on "Derivation of Axisymmetric Flexural Vibration Equations of a Cylindrically Aeolotropic Circular Plate"

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THOUGH the basic governing differential equations for a cylindrically aeolotropic plate subjected to bending and vibration have been developed by several researchers,[1-5] the axisymmetric flexural vibration equations and associated boundary conditions of a cylindrically aeolotropic circular plate of varying thickness have been ingeniously derived by Joung[6] using the method of variational calculus. As stated by Juong the advantage of this method is that both the differential equations and the appropriate boundary conditions can be obtained simultaneously.

In Joung's Note some errors should be noted and corrections made. As a result of the extremizations of the functional,

$$I = \int_{t_1}^{t_2} \iint_s f(t,r,w,w_r,w_t,w_{rr}) r dr d\theta dt \tag{1}$$

the differential equation of motion and the appropriate boundary conditions should read

$$D(c^2 w_{rrrr} + 2c^2 r^{-1} w_{rrr} - r^{-2} w_{rr} + r^{-3} w_r) + D_r[2c^2 w_{rrr} + (2c^2 + \nu) r^{-1} w_{rr} - r^{-2} w_r] + D_{rr}(c^2 w_{rr} + \nu r^{-1} w_r) + \mu w_{tt} = p(r,t) \tag{2}$$

$$[rD(c^2 w_{rr} + \nu r^{-1} w_r)\eta_r]_b{}^a = 0$$

and

$$\{[rD(c^2 w_{rrr} + c^2 r^{-1} w_{rr} - r^{-2} w_r) + rD_r(c^2 w_{rr} + \nu r^{-1} w_r)]\eta\}_b{}^a = 0 \tag{3}$$

For isotropic material, $c^2 = 1$, Eqs. (2) and (3) yield the results given by Eqs. (10) through Eqs. (13) in the Note. However, for the case of an annular plate of aeolotropic material with constant thickness, the differential equation should read

$$c^2 w_{rrrr} + 2c^2 r^{-1} w_{rrr} - r^{-2} w_{rr} + r^{-3} w_r + D^{-1} \mu w_{tt} = D^{-1} p(r,t) \tag{4a}$$

and the boundary conditions for simple support and free hole are

$$w = 0, \quad c^2 w_{rr} + \nu r^{-1} w_r = 0 \quad \text{at } r = a \tag{4b}$$

$$c^2 w_{rr} + \nu r^{-1} w_r = 0$$

and

$$c^2 w_{rrr} + c^2 r^{-1} w_{rr} - r^{-2} w_r = 0 \quad \text{at } r = b \tag{4c}$$

For static problems, Eq. (4a) reduces to the differential equation for the axisymmetric bending of the cylindrically aeolotropic plate, which is given by Carrier as Eq. (7) in Ref. 1.

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Incidentally, a typographical error appears both in Ref. 1 and Ref. 6. The definition of ν should be

$$\nu = -a_{12}/a_{22}.$$

References

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[4] Minkarah, I. A. and Hoppmann, W. H., II, "Flexural Vibration of Cylindrically Aeolotropic Circular Plates," *The Journal of the Acoustical Society of America*, Vol. 36, 1964, pp. 470–475.

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[6] Joung, K. S., "Derivation of Axisymmetric Flexural Vibration Equations of a Cylindrically Aeolotropic Circular Plate," *AIAA Journal*, Vol. 7, No. 7, July 1969, pp. 1386–1398.

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Comment on "Separation Solutions for Laminar Boundary Layer"

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IN Ref. 1 Fox and Saland report on the separation solutions for the compressible similar laminar boundary-layer equations with mass transfer. The reported results were limited to unit Prandtl number.

In Ref. 1 the governing boundary-value problem for the compressible similar laminar boundary layer with mass transfer [Eqs. (1, 2, 5, and 6) of Ref. 1] was integrated using a

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